

Multihomogeneous covariants and the essential dimension of finite groups

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The essential dimension, introduced by Buhler and Reichstein in 1995, is a numerical invariant attached to every finite group G measuring the complexity of the group G or more precisely of Galois G -algebras over extensions of a fixed base field k .

In this talk we will first investigate another invariant of finite groups, which is given by the least dimension of a faithful representation. This invariant, called representation dimension, is closely related to essential dimension, but conceptually simpler. We will study its behavior under various constructions that can be done with groups such as taking subgroups, direct products, wreath-products and group extensions.

Afterwards we shall go through the definition of the essential dimension of finite groups and check if this new invariant shows the same behavior under the above mentioned constructions as representation dimension. Essential dimension can be understood in geometric terms via so called covariants. A covariant of G is a G -equivariant rational map between (faithful) representations of G . The essential dimension of G asks for how much the G -action can be compressed by means of covariants.

The question arises naturally if among the covariants with the best compression there exist such which satisfy some useful property, e.g. being regular or being homogeneous. It turns out that for homogeneity this is only the case for a special class of group. We introduce a more general property, called multihomogeneity, and study multihomogeneous covariants in order to get new insights on essential dimension.